Morphogenesis : Basal Cognition :: Self-Organisation : Maximum Entropy

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1) Using the FEP we can understand any sort of system as performing elemental inference → referred to as 'basal cognition'

2) This has an interpretation as a gauge force acting on the system

What is constrained maximum entropy?

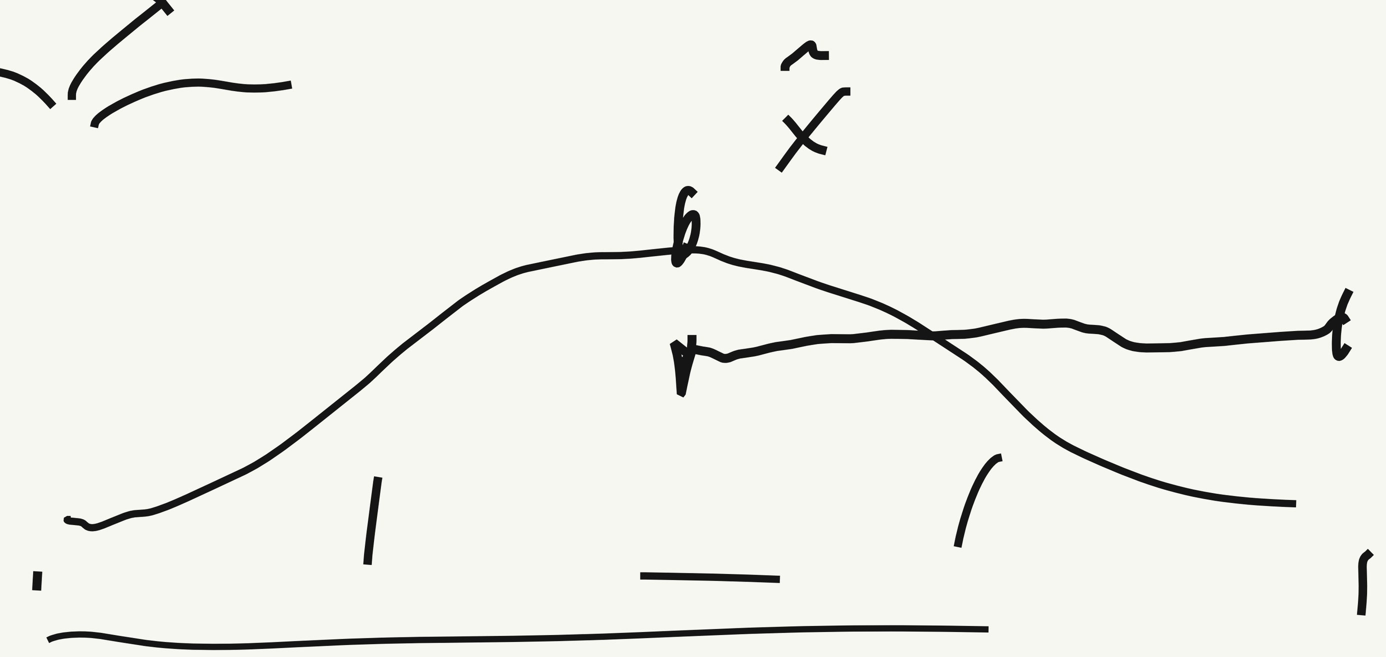
$$S[p; f] = -\int \ln p \, p - \lambda\left(\int f \, p + C\right)$$

$$f(x) = x$$

$$f(x) = \| x - \hat{x} \|$$

$$\lambda \int (x - \hat{x})^2 \, p(x) \, dx = C$$

$$\lambda \int x \, p(x) \, dx = \lambda C$$



$$\frac{1}{\lambda} \exp\left\{ \frac{1}{\lambda} x \right\}$$

$\eta ::$ ext.

$\mu ::$ int.

$b ::$ blanket.

$$p(\eta \mid b)$$

$$F = \langle E \rangle + TS$$

$$- \int p(\eta \mid \mu, b)\, q_\mu(\eta)\, d\eta + \int \ln q_\mu(\eta)\, q_\mu(\eta)\, d\eta$$

$$- \ln p(\mu, b)$$

μ s.t. $\sigma(\hat{\mu}_b) = \hat{\eta}_b$

$$p(\eta ; \hat{\eta} \mid b)$$

$$q(\eta ; \sigma(\hat{\mu}) \mid b)$$

$$=: \hat{\eta}$$

$$-\int \ln q_\mu(x)\, q_\mu(x)\, dx + \int \ln p(x \mid \mu, \sigma)\, q_\mu(x)\, dx - \ln p(\mu, \sigma)$$

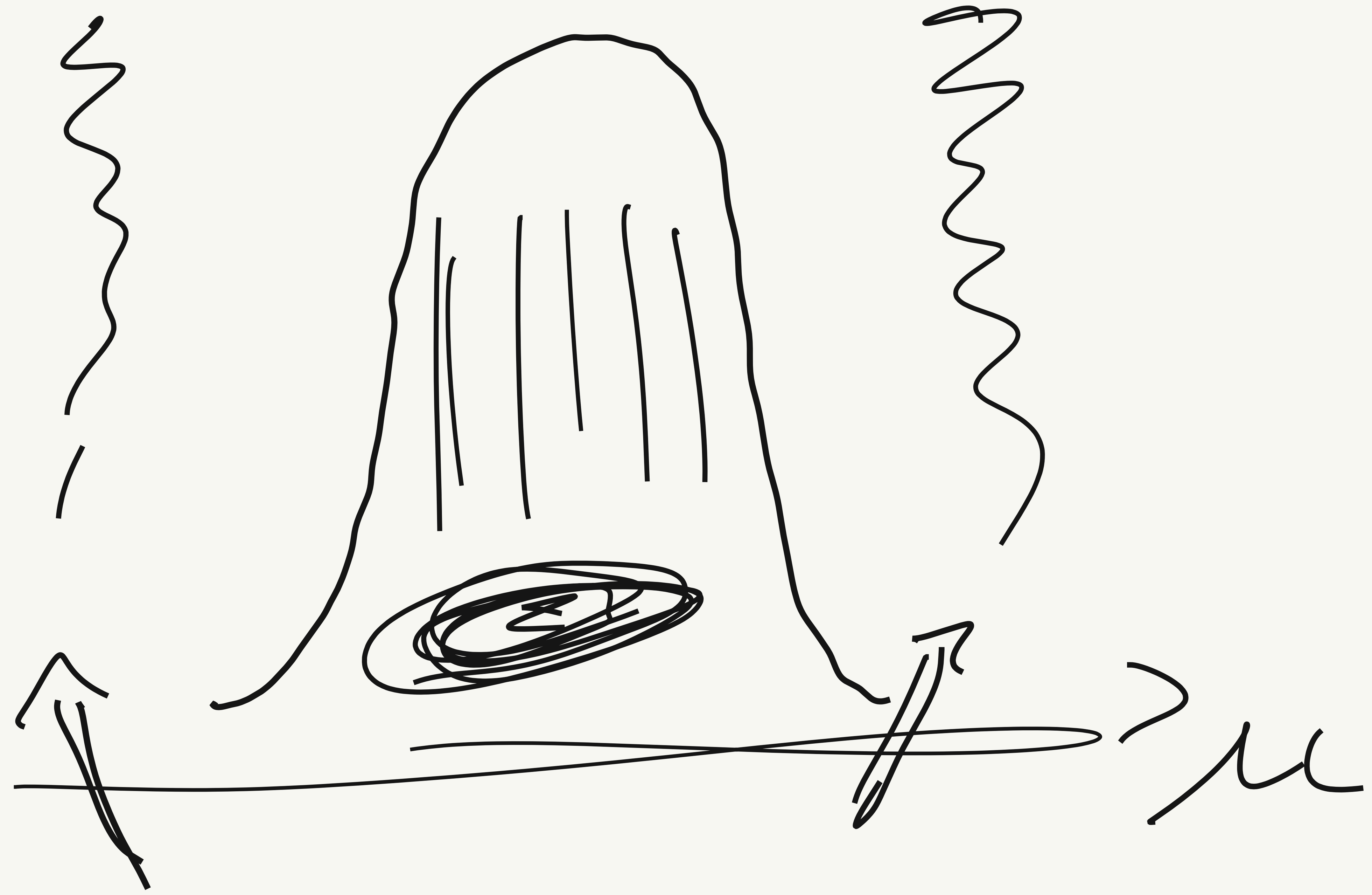
$$S[q] - = -\left(\int -\ln p(x \mid \mu, \sigma)\, q_\mu x\, dx + \ln p(\mu, \sigma) \right)$$

$$\int f(x)\, q = -\ln p(\mu, \sigma)$$

$$\int f(x)\, q + \ln p(\mu, \sigma)$$

$$f(x) = -\ln \underline{p}(x \mid \mu, \sigma)$$

$$-\int \ln \underline{p(x \mid \mu, \sigma)}\ \underline{q_\mu(x)}\, dx = \underline{-\ln p(\mu, \sigma)}$$

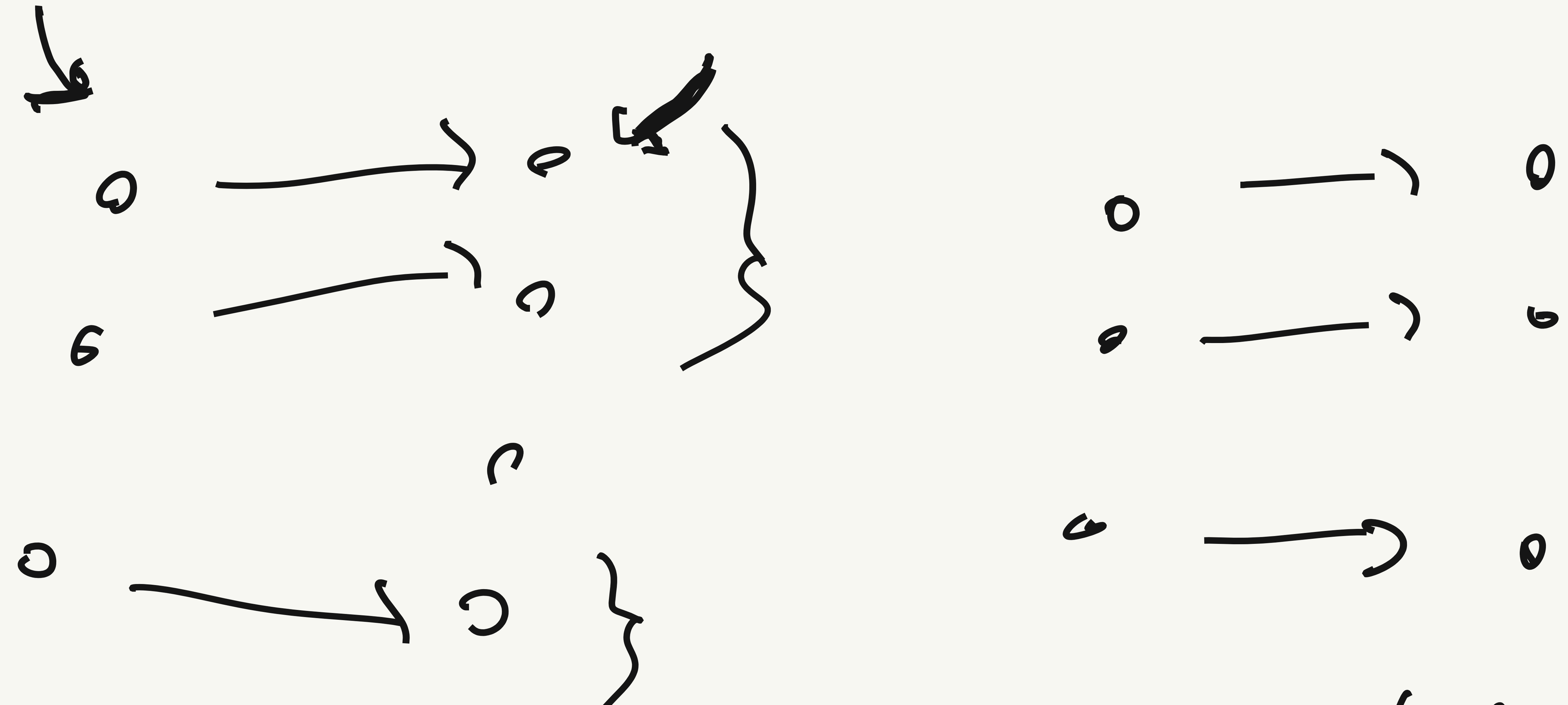


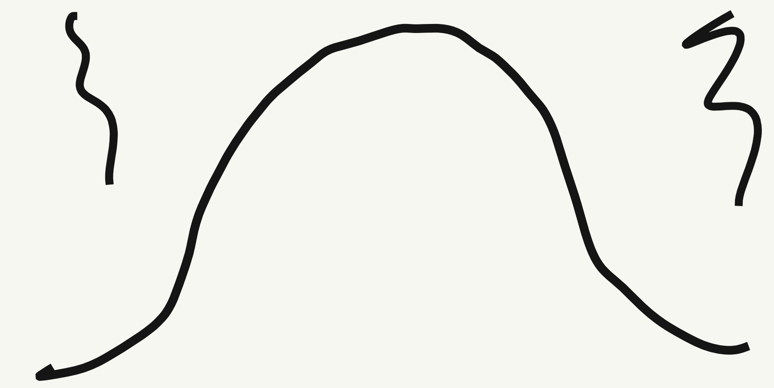
μ

$$p(n \mid \mu, b) = p(n \mid b)$$

$$\rightarrow p(\mu \mid n, b) = p(\mu \mid b)$$

$$\sigma(\mu_b) = n_b \rightsquigarrow \sigma(\mathbb{E}[\mu \mid b]) = \mathbb{E}[n \mid b]$$





$$\sigma^{-1}(\underbrace{\mathbb{E}[n \mid b]}_{\hat{n}_b}) = \mathbb{E}[\mu \mid b]$$

$$-\int \tilde{q}_n(\mu) \ln \tilde{q}_n(\mu) - \ldots$$

$$\int \mu \, \tilde{q}_n(\mu) \, d\mu = \sigma^{-1}(\hat{r}_b)$$

$$- S[\tilde{q}_n(\mu)] - \lambda \Big(- \int \ln p(\mu | b) \, p \, d\mu + \ln p(r, b) \Big)$$

$$\boxed{p(x) = Z e^{-\lambda J(x)}}$$

$$\tilde{q}_n(\mu) = e^{\cancel{-(-\ln p(\mu|b))}}$$

$$\underline{\underline{\tilde{q}_n(\mu) = p(\mu|b)}}$$

$$p(\mu ; \hat{\mu}_b | b)$$

$$:= e^{-\mu}$$

$$\tilde{q}_n(\mu) \propto e^{-\mu}.$$

$$J(\mu) = \mu.$$

$$q(\mu) = e^{-\mu}.$$

$$-\ln p - \lambda y = 0$$

$$p(x) \propto e^{-\lambda y}$$

$$-\cancel{\ln (e}^{-\lambda y}) - \lambda y$$

$$-\cancel{(-\lambda y)} - \lambda y = 0$$

$$\Rightarrow \quad -\int \ln p \, p \quad - \lambda \left(\int g\, p + C \right).$$

$$\mathbb{E}[g] = C \quad > 0$$

$$-\ln p \, p \quad - \lambda g \, p$$

$$-\ln p \quad - \lambda g \quad = 0$$

$$\leftarrow$$

$$e^{-\lambda g - \lambda g'} = e^{-\lambda g} e^{-\lambda g'}$$

$$S[p' : g'] = -\int \ln\{ e^{-\lambda g'} p \} e^{-\lambda g'} p \quad - \int (\lambda g' + \lambda g) e^{-\lambda g'} p$$

$$+ C$$

$$- \ln\{ e^{-\lambda g'} p \} e^{-\lambda g'} p \quad - (\lambda g' + \lambda g) e^{-\lambda g'} p + C$$

$$-\ln\{e^{-\lambda y'}p\}\, e^{-\lambda y'}p - (\lambda y' + \lambda y)\, e^{-\lambda y'}p + c$$

$$+\cancel{\lambda y'} - \ln p \; e^{-\lambda y'}p - (\cancel{\lambda y'} + \lambda y)\, e^{-\lambda y'}p$$

$$-\ln\{p\}\,\underline{e^{-\lambda y'}p} - \lambda y\,\underline{e^{-\lambda y'}p}$$

$$(-\ln\{p\} - \lambda y)\, e^{-\lambda y'}p$$

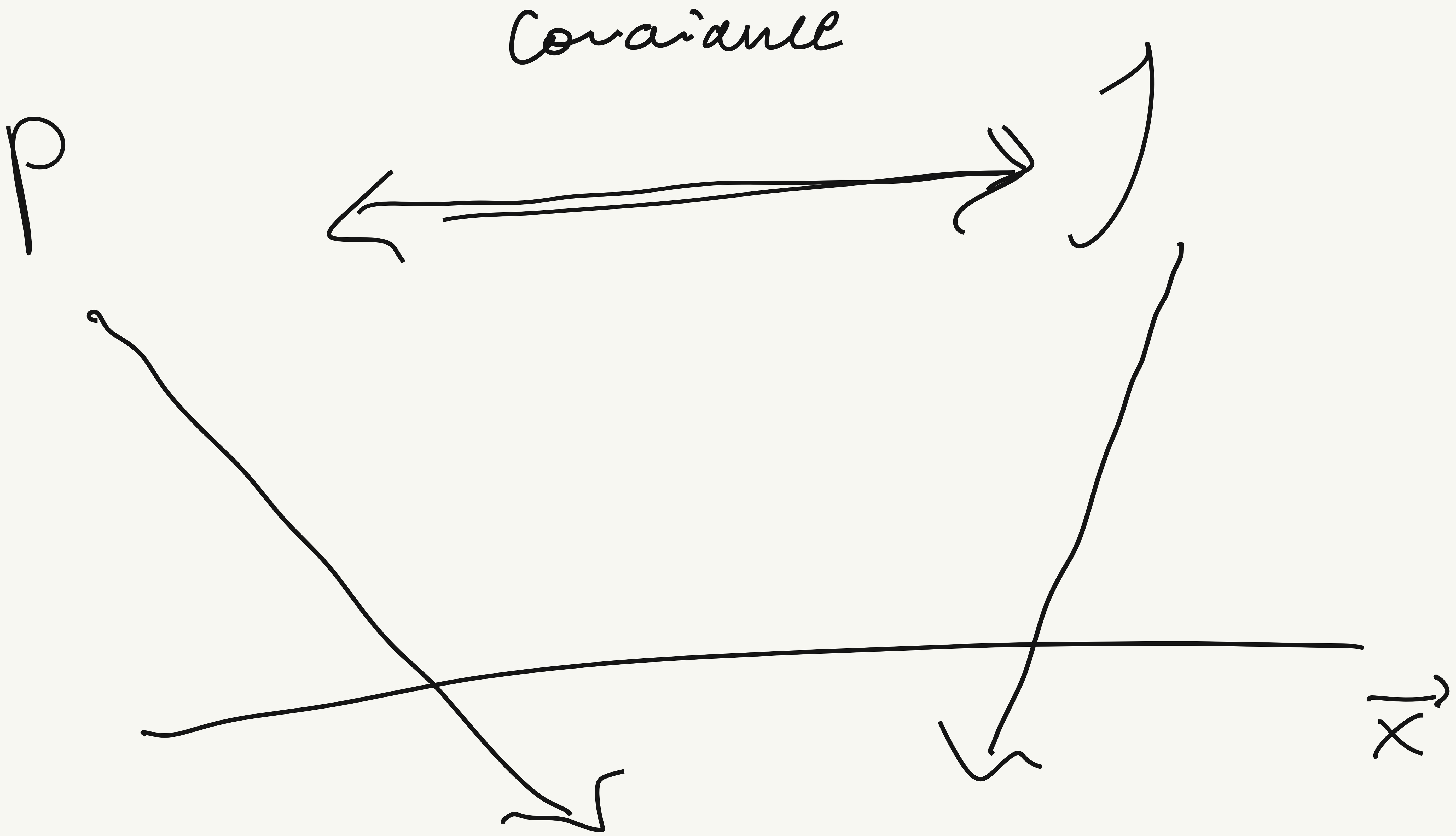
$$\int h(\phi)\, d\vec{x} \qquad \partial_\phi h(\phi) - \cancel{\frac{d}{dt}\partial_{\dot\phi} h(\phi)} = 0$$

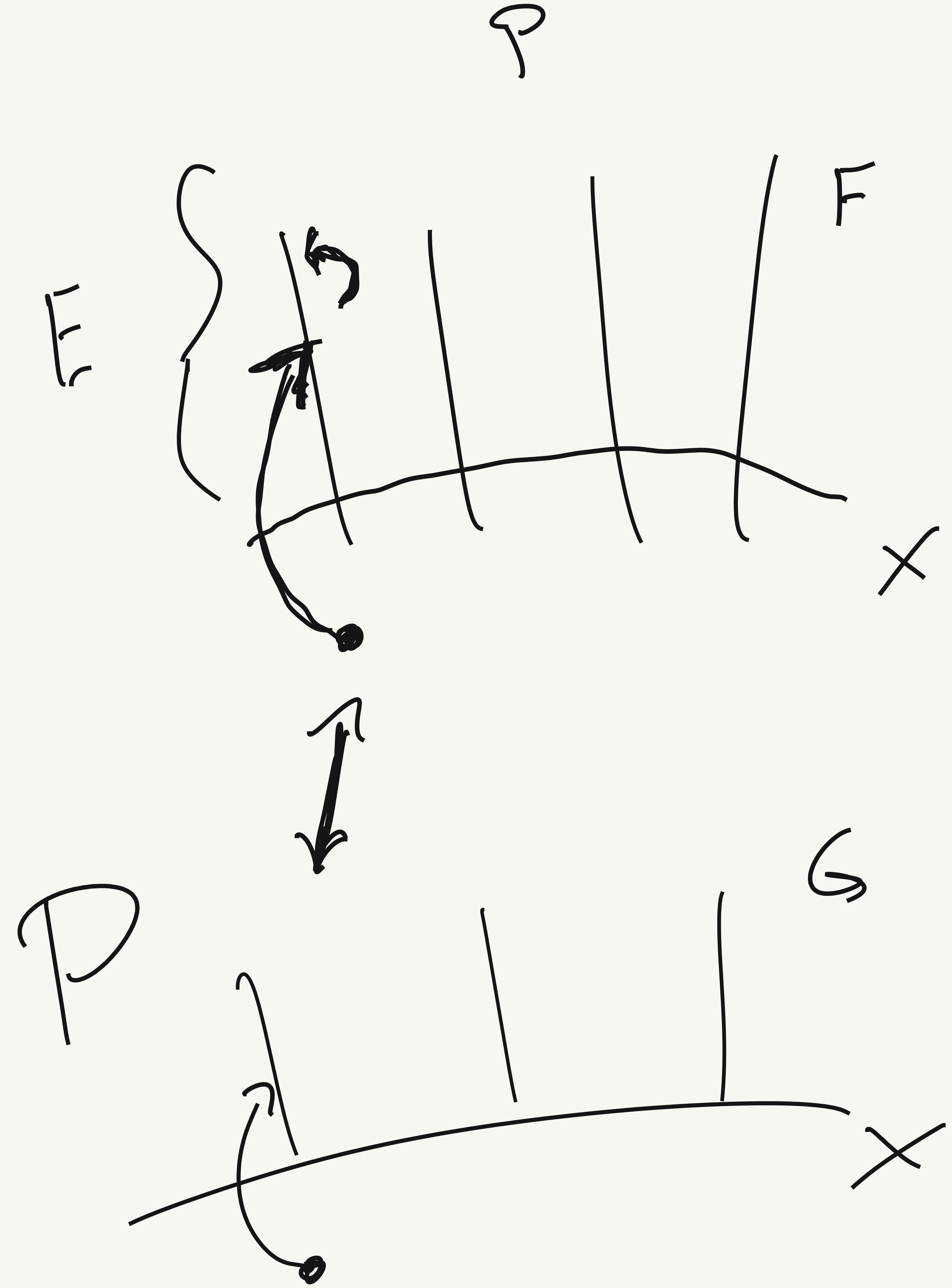
$$\partial_p h(p) = 0$$

$$\left(-\ln\{p\} - \lambda g\right) e^{-\lambda g'} p$$

$$-\ln p - \lambda g = 0$$

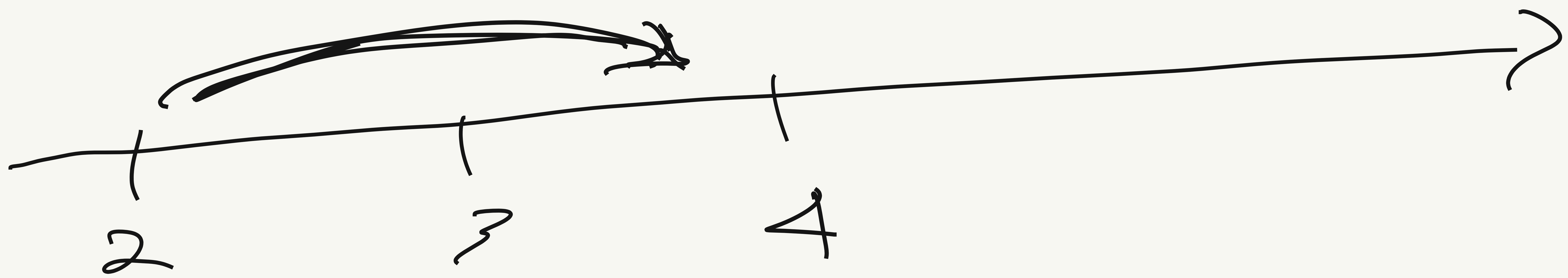
$$p = \exp\{-\lambda g\}$$





$P(x) = y \in \mathbb{R}$

1



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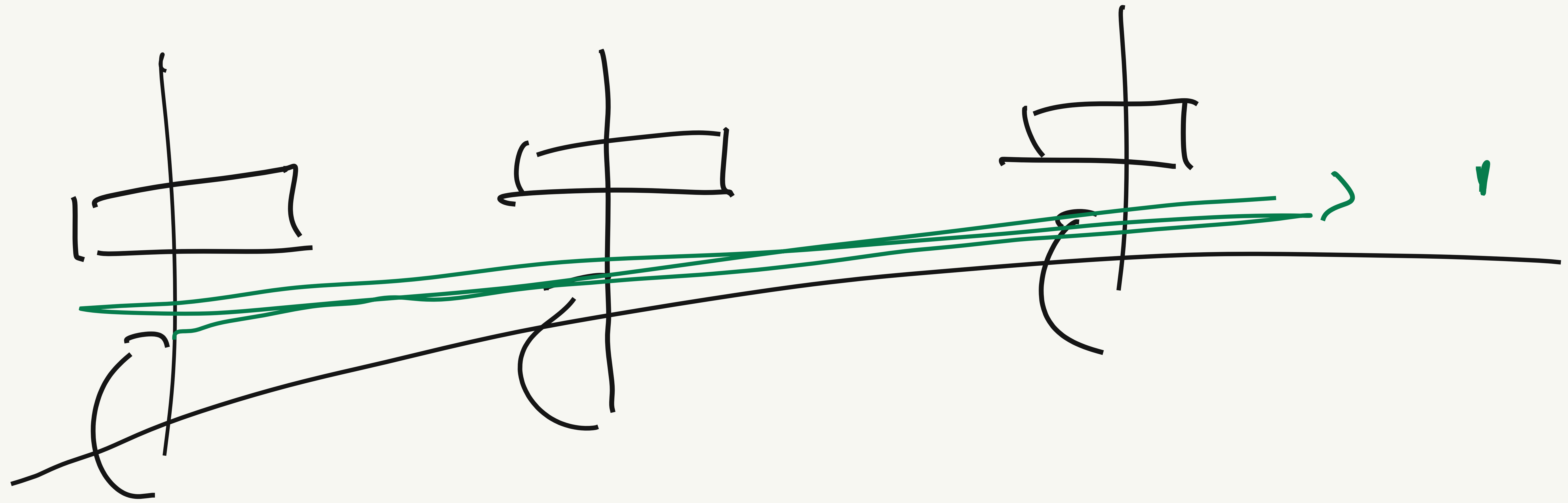
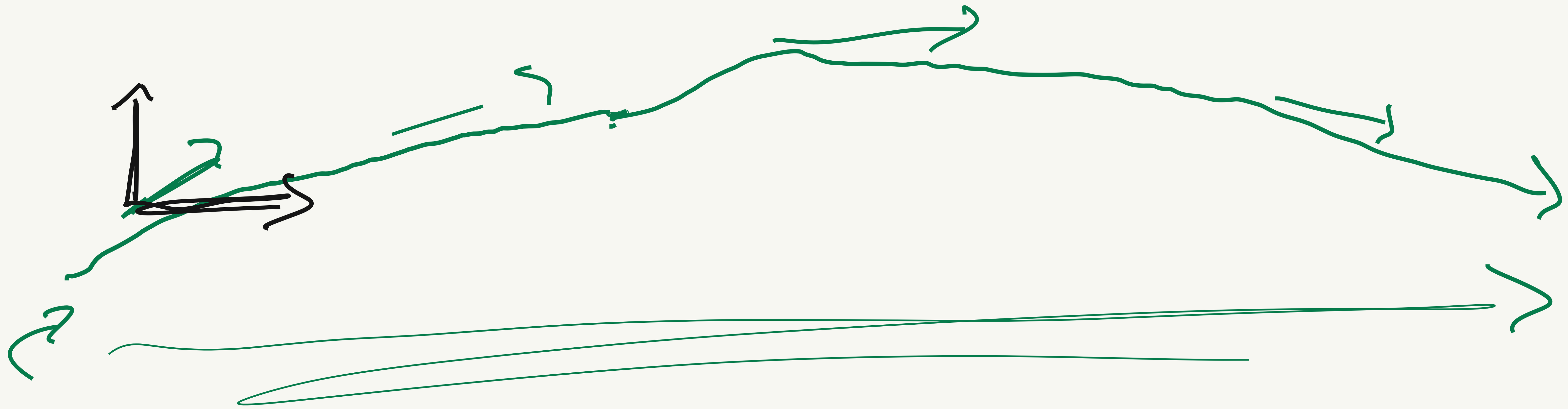
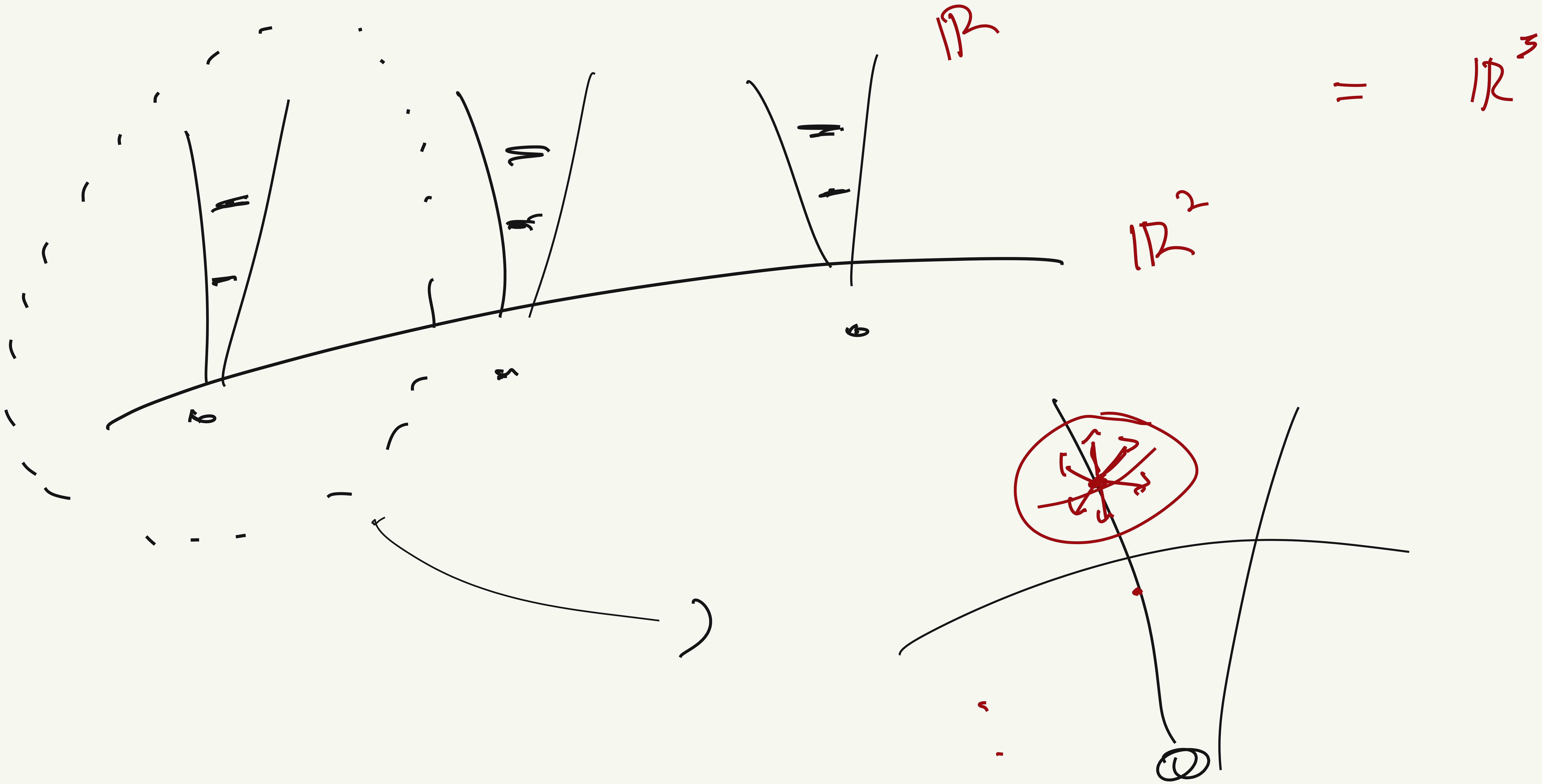
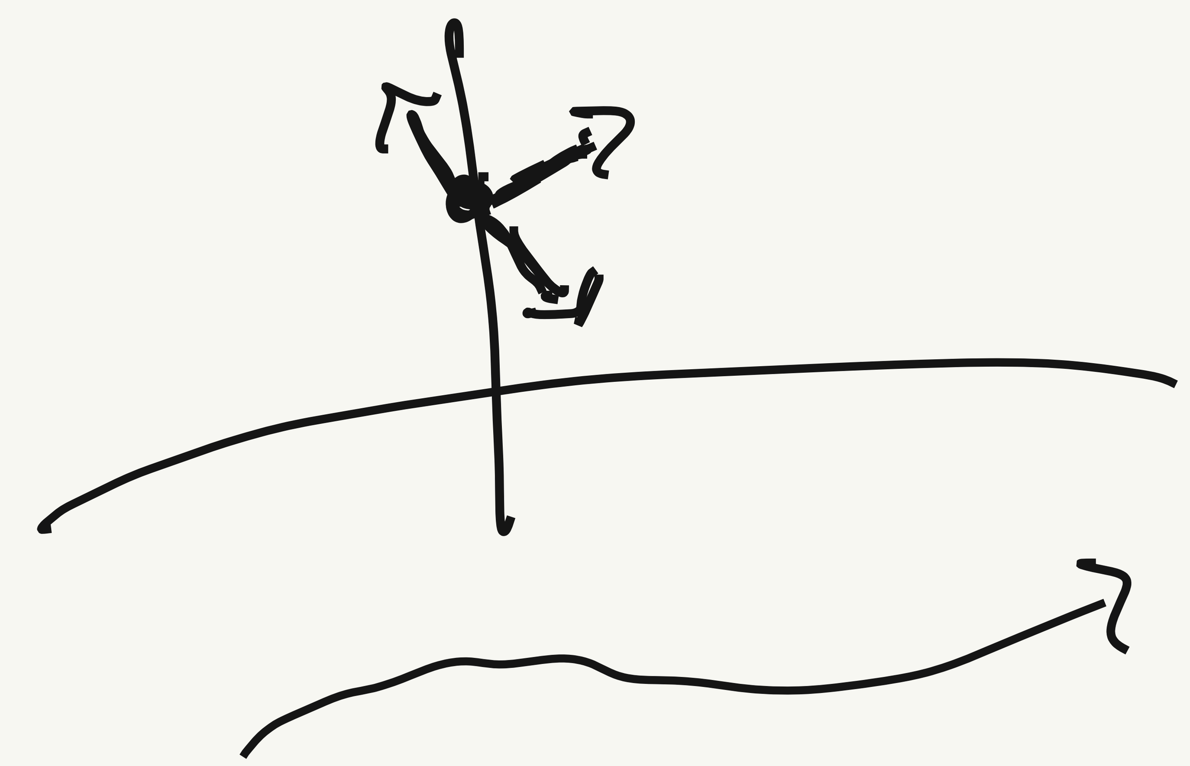
3
4

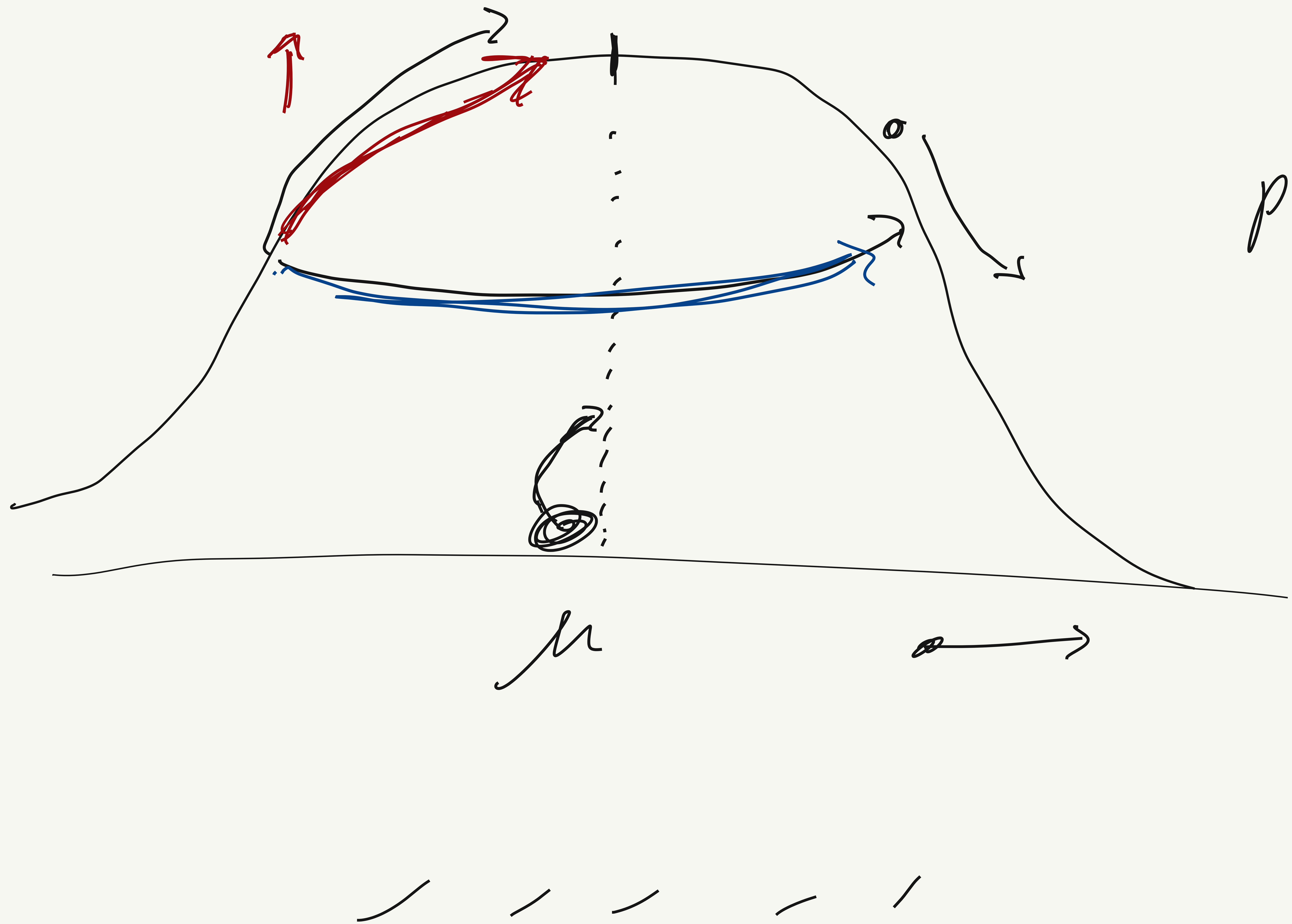








$\nabla f(x, y) = 2D$ vect.



$p(\mu \mid \sigma)$

$\sigma = 5$